

Dissipationless Disk Accretion

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We consider disk accretion resulting purely from the loss of angular momentum due to the outflow of plasma from a magnetized disk. In this limiting case, the dissipation due to the viscosity and finite electrical conductivity of the plasma can be neglected. We have obtained self-consistent, self-similar solutions for dissipationless disk accretion. Such accretion may result in the formation of objects whose bolometric luminosities are lower than the flux of kinetic energy in the ejected material.

I. INTRODUCTION

Disk-accretion mechanisms can provide clues about a number of processes occurring when matter is accreted onto a gravitating center. It is thus no wonder that such mechanisms have been intensely studied for more than 30 years. The detection of directed ejections from both the cores of active galaxies [1] and Galactic and intergalactic sources, such as SS433 [2], young stellar objects [3], and Galactic super luminal sources [4], has revealed an unambiguous connection between jets and accretion disks. The presence of a directed ejection in a source is essentially always associated with evidence for the presence of an accretion disk [3]. One exception is the jet like structures in plerions [5], whose nature is obviously different [6]. Thus, the collected observational data indicate that disk accretion is frequently (or even always) accompanied by the narrowly collimated ejection of a substantial fraction of the accreted matter from the source. This picture gives rise to a number of problems. What is the mechanism that leads to some fraction of the matter falling on to the gravitating center being ejected from the source? How is this matter collimated? What makes the collimated ejection stable enough to propagate to great distances from the source? What accelerates the charged particles that generate the observed radiation in the jets? This list of questions is far from complete. Here, we consider just one of them: what mechanism ejects some of the matter falling on to a gravitating center in the course of disk accretion? To fall onto a gravitating center, matter with nonzero angular momentum must somehow lose angular momentum. This angular momentum must be carried away by matter, so that disk accretion should inevitably be accompanied by an outflow of matter; the only problem is to determine its rate and direction. In the classical theory of . accretion disks [7], angular momentum of the accreted matter is lost via viscous, hydrodynamical, turbulent stresses. The classical theory does not include a magnetic field. In [8], however, the magnetic field was used to provide high viscosity. This study also considered the possibility of a matter outflow along the axis of rotation of the accretion disk in a hypercritical regime, when the radiation pressure exceeds the gravitation. A quantitative theory of such an outflow is given in [9], which shows that the outflow already begins in a state with subcritical luminosity. This ejection mechanism may be important in some sources with hyper-Eddington luminosities; however, it is clear that, in many cases (e.g., in young stellar objects), the ejection is driven by another factor, since the radiation pressure in these objects is too small to support this mechanism. Even in SS433, the ejection energy cannot be provided purely by radiation pressure [10]. In standard disk-accretion theory, angular momentum is lost via viscous stresses due to the differential rotation of the disk. Radiation-efficient disks are geometrically thin, with a local temperature that is substantially lower than the virial temperature. It is very difficult to provide an appreciable outflow from the disk under these conditions, since the disk matter is in a very deep potential well. Recently, accretion in radiatively inefficient disks (ADAFs) for which the ion temperature is close to the virial temperature have been widely discussed [13, 14]. Generally speaking, the matter in such disks can flow out vigorously, with the subsequent formation of a jet. However, the existence of ADAF disks is in doubt [15]. Therefore, it remains important to search for alternative mechanisms for the ejection of accreted matter. Ejections due to the presence of a magnetic field in the disks are of particular interest.

Blandford and Payne [16] have suggested one of the most promising mechanisms for jet outflows. Matter moves in the disk with close to Keplerian velocities. If a magnetic field penetrates the disk, the matter is blown off the disk by centrifugal forces if the inclination of the magnetic-field lines to the disk is smaller than 60° . This mechanism is particularly easy to understand if we imagine the motion of the plasma along a magnetic line to be like that of a bead on a rotating wire. The bead moves in the disk along a circular Keplerian orbit, with the sum of all the forces being zero. If a field line is inclined to the equator at an angle smaller than 60° , any shift of the bead along the magnetic-field line will result in an increase in the centrifugal force and a decrease in the gravitation force. In this case, the position of the bead on the disk is unstable. In addition to forming an ejection, this mechanism can explain the loss of angular momentum, which is carried away from the disk by the wind. The influence of the matter outflow on the dynamics of the accretion disks themselves was considered in [17–19] in the context of young stellar objects. It was shown that the loss of angular momentum due to the wind from the disk can be more efficient than losses due to turbulent viscosity. This raises the issue that disk accretion onto a gravitating center should take into account the

fact that some of the angular momentum is carried away by the disk wind. Ferreira [21, 22] attempted to solve this problem. His formulation is similar to that used previously by Bisnovatyi-Kogan and Ruzmaikin [20] in one of their pioneering studies on the theory of disk accretion in a magnetized plasma. It is assumed that the interstellar plasma is the source of an almost uniform magnetic field. In this case, in order for matter to fall onto the gravitating center, not only must angular momentum be lost, but matter must also diffuse across the magnetic field. In addition to a high turbulent viscosity, this requires a low electrical conductivity of the plasma, which must be associated with a high level of turbulence. In other words, dissipation plays a key role in this case. We will use another formulation of the problem. Figure 1a presents the difference between our formulation and that used by Ferreira et al. [21, 22] (left). Interstellar matter with some initial angular momentum and a relatively uniform magnetic field is accreted onto a gravitating center. The accreted matter diffuses across the field lines, while the forming wind flows out along magnetic lines stretched from interstellar matter. Figure 1b (right) presents another situation. Matter is accreted onto the compact object along with the frozen-in magnetic field, forming the accretion disk. During the accretion, magnetic lines emerge from the accretion disk and are stretched. A specific mechanism for this stretching of magnetic lines from the disk was proposed and studied numerically in [23]. It results in the formation of a corona and, accordingly, a magnetized disk wind. In this case, the accretion disk itself is a source of the magnetic field of the outflowing wind, just as the Sun is the source of the magnetic field in the solar wind. Note that precisely such a formulation of the problem was used in the study [16] of the centrifugal mechanism for the outflow of matter from an accretion disk. Here, we study not only the outflow itself, as was done in [16], but also the influence of the plasma outflow from an accretion disk on the accretion. This is not only important for the physics of accretion disks, but is also directly applicable to the interpretation of the available observational data. Observations of Galactic superluminal sources [4, 24] are of particular interest, since they provide conclusive evidence that plasma ejections substantially affect the accretion process.

II. FORMULATION OF THE PROBLEM

In general, viscous stresses in accretion disks are undoubtedly important for angular-momentum transport. Here, we primarily consider the influence of the wind on the accretion dynamics. Therefore, we will consider the limiting case when all of the angular momentum is carried away from the disk by the wind with no dissipation. The possibility of such dissipationless accretion can easily be understood based on the following simple consideration. Let one of the components of a binary rotate around a fixed gravitating center (Fig. 2). In the absence of dissipation and wind, this rotation will continue indefinitely. Let us now suppose that a magnetized wind flows from the rotating object, and that the Alfvén radius is R_a . We will assume that the wind is spherically symmetrical and its velocity is v_0 . For simplicity, we will use here the mechanical analogy of beads moving along wires for the motion of the plasma in a strong magnetic field. This kind of motion occurs up to the Alfvén radius, beyond which the plasma moves essentially freely. We assume that there is no intrinsic rotation of the object, so that its angular momentum is $M \cdot l_z$, where M is the mass of the object, and $l_z = \Omega R_0^2$ is the specific angular momentum due to its orbital motion. We denote L to be the total angular momentum contained in a sphere with radius R_a with its center at the point $\mathbf{R}_0(t)$ at time t . At time $t + \delta t$, the angular momentum contained in this sphere (with its center at $\mathbf{R}_0(t)$), will differ from L . The equation for the variation of the angular momentum has the form

$$\frac{dL}{dt} = - \oint \rho [\mathbf{r}_a \mathbf{v}]_z v_n dS. \quad (1)$$

Here ρ is the density of the plasma, v_n is the velocity normal to the integration surface, the integration is carried out over the Alfvén surface, and the z axis is directed along the angular-momentum vector. We can see from Fig. 2 that $\mathbf{r}_a = \mathbf{R}_0 + \mathbf{R}_a$, the velocity of the plasma is $\mathbf{v} = v_0 \mathbf{n} + [\boldsymbol{\Omega} \mathbf{r}_a]$, where $\mathbf{n} = \mathbf{R}_a/R_a$ is the unit vector in the direction \mathbf{R}_a , and $\boldsymbol{\Omega}$ is the angular-velocity vector for the rotation about the fixed center. After integrating over the directions of the vector \mathbf{n} , we obtain

$$\frac{dL}{dt} = -\frac{2}{3} \dot{M} \Omega (2 R_0^2 + R_a^2). \quad (2)$$

To interpret this result, note that $\dot{L}_M = -\dot{M} \Omega R_0^2$ is the variation of the angular momentum due to the decrease of the mass of the rotating object. It can be shown that the rate of variation of the angular momentum of the wind is $\dot{L}_V = -\dot{M} \Omega R_0^2/3$. This can be determined as the difference between the angular momenta $L_V(t + \delta t)$ and $L_V(t)$ of the wind particles in a sphere of radius R_a with its center at the fixed point $\mathbf{R}_0(t)$. The sum $\dot{L}_M + \dot{L}_V$ yields the first term in (2). Therefore, the second term describes the rate of variation of the specific angular momentum associated

with the orbital motion:

$$\frac{dl_z}{dt} = -\frac{2}{3} \frac{\dot{M}}{M} \Omega R_a^2. \quad (3)$$

Thus, the outflowing wind carries away the angular momentum of the rotating object.

This simple example illustrates an important property of accretion in the presence of a magnetized wind. Since the accretion is not accompanied by heating, the wind carries away not only angular momentum, but also rotational energy. Dissipationless accretion can result in the formation of ejections for a relatively low luminosity of the object. This makes this accretion mechanism particularly interesting: it can explain the unusually high efficiency for the transformation of the gravitational energy of the accreted matter into the kinetic energy of the jet in the unusual source SS433. The bolometric luminosity of SS433 is about $3 \cdot 10^{39}$ erg/s [25], while the flux of kinetic energy from the object is at least twice this value ($> 6 \cdot 10^{39}$ erg/s) [12], which is difficult to understand in classical accretion models [10]. It is reasonable to consider dissipationless accretion in an ideal magnetohydrodynamical (MHD) approximation. This leads to the model in the left panel of Fig. 3: the wind stretches magnetic-field lines from the accretion disk. The polarity of the lines varies chaotically, since the total flux of the magnetic field emerging from either side of the accretion disk is, on average, zero. At first glance, it would seem impossible to describe the dynamics of a wind in the presence of a magnetic field with arbitrarily varying polarity at the disk surface. However, there is a circumstance that simplifies the situation radically. As was noted in [26], the dynamics of an ideal plasma are invariant with respect to the direction of the magnetic-field lines. Therefore, if the direction of the field lines varies so that the polarity of the magnetic field is the same on each side of the accretion disk, this does not affect the plasma motion, and we are able to consider a self-consistent outflow of plasma from an accretion disk with an azimuthally symmetrical magnetic field, shown schematically in Fig. 3 (right). Two features make this approach different from the well-known formulation of Blandford and Payne [16]. First, the outflowing wind must be consistent with the accretion rate in the disk, whereas, in [16], the disk and wind parameters were specified independently. Second, since the plasma moves toward the gravitating center, there is a nonzero azimuthal electrical field, $E_\varphi \neq 0$. Note that a similar formulation was given by Contopoulos [27]. However, his study was not continued, since it was clear that, in this case, magnetic flux should accumulate at the center. If the polarity of the field is the same everywhere, it cannot be annihilated, and the accretion will inevitably be halted by the pressure of the magnetic field. As we noted above, this problem is removed because we are solving for a plasma flow in the presence of a magnetic field of variable polarity. Therefore, the magnetic flux does not accumulate at the center, and is always equal to zero, on average. Since the magnetic-field lines possess different polarities, they can reconnect, so that the magnetic-field pressure at the center will not increase without bound. Finally, we will assume that the plasma flow in the wind is self-similar. This assumption is natural if we are interested in the motion of plasma directly above a very thin disk away from its edges. Then, the problem has no parameters with the dimension of length, and the flow becomes self-similar. This approximation has been used to describe magnetized winds from accretion disks in numerous studies, starting with [16]. Vlahakis and Tsinganos [28] present a detailed description and classification of all possible types of selfsimilarity. Note also the study [29], which gives selfsimilar solutions describing nonstationary accretion.

III. GENERAL RELATIONS

The dimensionless steady-state equations for an ideal, cool plasma (with pressure $P = 0$) can be written (see, for example, [30])

$$\rho(\mathbf{v} \nabla) \mathbf{v} = -\frac{1}{2} \nabla B^2 + (\mathbf{B} \nabla) \mathbf{B} - \rho \frac{g \mathbf{R}}{R^3}, \quad (4)$$

$$\text{div}(\rho \mathbf{v}) = 0, \quad (5)$$

$$\text{div} \mathbf{B} = 0, \quad (6)$$

$$\text{curl}[\mathbf{v} \times \mathbf{B}] = 0. \quad (7)$$

Here, the coordinates, density, velocity, and magnetic field are expressed in units of their characteristic values R_0 , ρ_0 , v_0 , B_0 , respectively; in addition,

$$\rho_0 v_0^2 = \frac{B_0^2}{4\pi}. \quad (8)$$

The parameter

$$g = GM / (R_0 v_0^2) , \quad (9)$$

where G the gravitational constant and M the mass of the central object.

Let us consider azimuthally symmetrical solutions for these equations. The dimensioned variables will be chosen as follows. Let us arbitrarily select a field line, and adopt R_0 as the distance from the center to the point where this line crosses the disk. We will take v_0 to be the Keplerian velocity of a particle at a distance R_0 from the center. We denote $2\pi\Psi_0$ to be the magnetic flux through a circle with radius R_0 located in the disk and with its center on the axis of symmetry. B_0 can then be defined by the relation $B_0 = \Psi_0/R_0^2$. The dimensioned density $B_0 = \Psi_0/R_0^2$ is defined by relation (8); $g = 1$. Below, we will use only dimensionless variables.

In the azimuthally symmetrical case, the field lines form a two-parameter family of curves, which can be presented in the form

$$\mathbf{R}(\alpha, \psi, \varphi) = (r(\alpha, \psi) \cos \phi, r(\alpha, \psi) \sin \phi, z(\alpha, \psi)) , \quad (10)$$

where $\phi = \varphi + \eta(\alpha, \psi)$. The parameters ψ, φ specify a field line, and α varies along a given line ($\mathbf{B} \sim \partial\mathbf{R}/\partial\alpha$). The substitution $\varphi \rightarrow \varphi - \varphi_0$ is equivalent to a rotation by φ_0 of the field line as a whole. The functions of two variables $r(\alpha, \psi)$, $z(\alpha, \psi)$, and $\eta(\alpha, \psi)$ are to be determined from the solution of the MHD equations. Following [31, 32], we will call the parameters α, ψ , and φ the frozen-in coordinates. The cylindrical coordinates (r, ϕ, z) can be expressed in terms of the frozen-in coordinates by the formulas

$$r = r(\alpha, \psi), \quad \phi = \varphi + \eta(\alpha, \psi), \quad z = z(\alpha, \psi).$$

Let us make a transformation from Cartesian to frozen-in coordinates in (4)(7). J denotes the Jacobian of the transformation:

$$J = \frac{\partial(X, Y, Z)}{\partial(\alpha, \psi, \varphi)} = \mathbf{R}_\alpha [\mathbf{R}_\psi \mathbf{R}_\varphi] , \quad (11)$$

where the subscripts denote differentiation with respect to the corresponding variables ($\mathbf{R}_\alpha \equiv \partial\mathbf{R}/\partial\alpha, \dots$). Since field lines do not cross, there is a one-to-one relation between the frozen-in and Cartesian coordinates, so that the Jacobian J never vanishes. Note the following useful equalities, which can be used to easily change between Cartesian and frozen-in coordinates:

$$\mathbf{R}_\alpha \nabla = \frac{\partial}{\partial\alpha}, \quad \mathbf{R}_\psi \nabla = \frac{\partial}{\partial\psi}, \quad \mathbf{R}_\varphi \nabla = \frac{\partial}{\partial\varphi}. \quad (12)$$

and also

$$\frac{\mathbf{R}_\alpha}{J} = [\nabla\psi, \nabla\varphi], \quad \frac{\mathbf{R}_\psi}{J} = [\nabla\varphi, \nabla\alpha], \quad \frac{\mathbf{R}_\varphi}{J} = [\nabla\alpha, \nabla\psi], \quad (13)$$

and

$$\frac{[\mathbf{R}_\psi, \mathbf{R}_\varphi]}{J} = \nabla\alpha, \quad \frac{[\mathbf{R}_\varphi, \mathbf{R}_\alpha]}{J} = \nabla\psi, \quad \frac{[\mathbf{R}_\alpha, \mathbf{R}_\psi]}{J} = \nabla\varphi. \quad (14)$$

The operator ∇ in frozen-in coordinates has the form

$$\nabla = \frac{1}{J} \left\{ [\mathbf{R}_\psi \mathbf{R}_\varphi] \frac{\partial}{\partial\alpha} + [\mathbf{R}_\varphi \mathbf{R}_\alpha] \frac{\partial}{\partial\psi} + [\mathbf{R}_\alpha \mathbf{R}_\psi] \frac{\partial}{\partial\varphi} \right\}. \quad (15)$$

Suppose that the magnetic field strength is specified by the equality

$$\mathbf{B} = \frac{\mathbf{R}_\alpha}{J} = [\nabla\psi, \nabla\varphi]. \quad (16)$$

In this case, (6) becomes an identity (it essentially expresses the field in terms of the Euler potentials). Expression (16) implies that we take the magnetic flux (in units of $2\pi\Psi_0$) through a circle with radius r as the frozen-in coordinate ψ . Note that the ratio \mathbf{R}_α/J is invariant with respect to the reparameterization $\alpha \rightarrow \alpha' = g(\alpha, \psi, \varphi)$; i.e.,

$$\frac{\mathbf{R}_\alpha}{J} = \frac{\mathbf{R}_{\alpha'}}{J'} ,$$

where $g(\alpha, \psi, \varphi)$ is an arbitrary function and $J' = \partial(X, Y, Z)/\partial(\alpha', \psi, \varphi) = J/g_\alpha$. This can be used to assign an arbitrary value to the Jacobian J without varying the field strength. It is convenient to relate J with the plasma density:

$$J = 1/\rho, \quad (17)$$

in which case the magnetic field is

$$\mathbf{B} = \rho \mathbf{R}_\alpha. \quad (18)$$

Let us introduce the three mutually orthogonal unit vectors

$$\mathbf{e}^{(r)} = (\cos \phi, \sin \phi, 0), \quad \mathbf{e}^{(\phi)} = (-\sin \phi, \cos \phi, 0), \quad \mathbf{e}^{(z)} = (0, 0, 1). \quad (19)$$

We obtain from (10)

$$\left. \begin{aligned} \mathbf{R}_\alpha &= r_\alpha \mathbf{e}^{(r)} + r \eta_\alpha \mathbf{e}^{(\phi)} + z_\alpha \mathbf{e}^{(z)}, \\ \mathbf{R}_\psi &= r_\psi \mathbf{e}^{(r)} + r \eta_\psi \mathbf{e}^{(\phi)} + z_\psi \mathbf{e}^{(z)}, \\ \mathbf{R}_\varphi &= r \mathbf{e}^{(\phi)}. \end{aligned} \right\} \quad (20)$$

Therefore, the Jacobian of the transformation is

$$J = r (z_\alpha r_\psi - z_\psi r_\alpha), \quad (21)$$

and, in accordance with (17), the plasma density is

$$\rho = \frac{1}{r (z_\alpha r_\psi - z_\psi r_\alpha)}. \quad (22)$$

Let us expand the plasma velocity into three noncoplanar vectors:

$$\begin{aligned} \mathbf{v} &= f \mathbf{R}_\alpha + \Omega \mathbf{R}_\varphi + \varepsilon \mathbf{R}_\psi = \\ &= (f r_\alpha + \varepsilon r_\psi) \mathbf{e}^{(r)} + (f z_\alpha + \varepsilon z_\psi) \mathbf{e}^{(z)} + r (f \eta_\alpha + \varepsilon \eta_\psi + \Omega) \mathbf{e}^{(\phi)}, \end{aligned} \quad (23)$$

where the functions f, Ω, ε may depend on the frozen-in coordinates. It turns out that, in both the stationary and nonstationary cases [31, 32], Eqs. (5) and (7) in frozen-in coordinates can be integrated in general form. Substituting (23) and (17) into the continuity equation (5) and transforming to frozen-in coordinates, we obtain

$$\frac{\partial f}{\partial \alpha} + \frac{\partial \Omega}{\partial \varphi} + \frac{\partial \varepsilon}{\partial \psi} = 0. \quad (24)$$

The electrical-field strength is

$$\mathbf{E} = -\frac{1}{c} [\mathbf{v} \mathbf{B}] = \frac{1}{c} (\varepsilon \nabla \varphi - \Omega \nabla \psi). \quad (25)$$

It follows that (7) in frozen-in coordinates has the form

$$\text{curl}[\mathbf{v} \mathbf{B}] = [\nabla \Omega \nabla \psi] - [\nabla \varepsilon \nabla \varphi] = \rho \left(\frac{\partial \Omega}{\partial \alpha} \mathbf{R}_\varphi + \frac{\partial \varepsilon}{\partial \alpha} \mathbf{R}_\psi - \left(\frac{\partial \Omega}{\partial \varphi} + \frac{\partial \varepsilon}{\partial \psi} \right) \mathbf{R}_\alpha \right) = 0. \quad (26)$$

The derivative $\partial \Omega / \partial \varphi$ due to the assumed azimuthal symmetry. Solving (24) and (26), we find that $\varepsilon = \text{const}$, while the functions f and Ω may depend only on the single variable ψ . In this case, (5) and (7) are identically satisfied. The gradient $\nabla \psi$ has only r and z components; the first term in (25) specifies the azimuthal component of the electrical field:

$$E_\phi = \frac{\varepsilon}{cr}. \quad (27)$$

The velocity of the accretion disk \mathbf{u} is also expressed in frozen-in coordinates. Let us suppose that the disk surface corresponds to the parameter $\alpha = 0$; i.e., $z(0, \psi) = 0$. Then, the derivative $z_\psi(0, \psi) = 0$. The plasma density above the disk surface is

$$\rho_0 = \frac{1}{r z_\alpha r_\psi} \Big|_0, \quad (28)$$

The z component of the magnetic field is

$$B_{z0} = \rho z_\alpha \Big|_0 = \frac{1}{r r_\psi} \Big|_0. \quad (29)$$

The radial component of the electrical field at the disk surface is

$$E_{r0} = -\frac{1}{c} [\mathbf{v} \mathbf{B}]_r \Big|_0 = -\frac{1}{r_\psi c} (\Omega + \varepsilon \eta_\psi) \Big|_0, \quad (30)$$

while the azimuthal component is defined by (27). Using the frozen-in condition

$$\mathbf{E} = -\frac{1}{c} [\mathbf{u} \mathbf{B}],$$

the continuity of the z component of the magnetic field at the boundary between the plasma and disk, and the continuity of the r and ϕ components of the electrical field, we obtain the components of the disk velocity

$$u_r = \varepsilon r_\psi \Big|_0, \quad u_\phi = r (\Omega + \varepsilon \eta_\psi) \Big|_0. \quad (31)$$

After transforming to frozen-in coordinates, the Euler equation (4) assumes the form

$$\begin{aligned} & \left(f \frac{\partial}{\partial \alpha} + \Omega \frac{\partial}{\partial \varphi} + \varepsilon \frac{\partial}{\partial \psi} \right) (f \mathbf{R}_\alpha + \Omega \mathbf{R}_\varphi + \varepsilon \mathbf{R}_\psi) - \frac{\partial}{\partial \alpha} (\rho \mathbf{R}_\alpha) = \\ & = -\frac{1}{2} \left\{ [\mathbf{R}_\psi \mathbf{R}_\varphi] \frac{\partial}{\partial \alpha} + [\mathbf{R}_\varphi \mathbf{R}_\alpha] \frac{\partial}{\partial \psi} + [\mathbf{R}_\alpha \mathbf{R}_\psi] \frac{\partial}{\partial \varphi} \right\} (\rho \mathbf{R}_\alpha)^2 - g \frac{\mathbf{R}}{R^3}. \end{aligned} \quad (32)$$

Substituting \mathbf{R} the form (10) and ρ in the form (22), we can obtain equations for the functions $r(\alpha, \psi)$, $z(\alpha, \psi)$, and $\eta(\alpha, \psi)$.

IV. BOUNDARY CONDITIONS

Let us study the relation between the field components and the parameters of the accretion disk, assuming the disk is infinitely thin (which is valid for a cool plasma). We will specify the volume density of the disk in the form

$$\rho_d(r, z) = \sigma(r) \delta(z), \quad (33)$$

where $\delta(z)$ is a delta function and $\sigma(r)$ is the mass surface density. Equation (4) is valid for $z > 0$ (or $z < 0$); i.e., beyond the disk. The dynamics of the plasma in the disk are specified by the equation

$$\rho_d(\mathbf{u} \nabla) \mathbf{u} + \rho(\mathbf{v} \nabla) \mathbf{v} = -\frac{1}{2} \nabla \mathbf{B}^2 + (\mathbf{B} \nabla) \mathbf{B} - (\rho_d + \rho) \frac{g \mathbf{R}}{R^3}. \quad (34)$$

We will integrate (34) over z in the small interval $(-z_0, z_0)$ and take the limit $z_0 \rightarrow +0$. This operation will be applied to each term of the equation. In cylindrical coordinates,

$$\nabla = \mathbf{e}^{(r)} \frac{\partial}{\partial r} + \mathbf{e}^{(z)} \frac{\partial}{\partial z} + \mathbf{e}^{(\varphi)} \frac{\partial}{r \partial \phi}, \quad (35)$$

so that

$$\begin{aligned} \rho_d(\mathbf{u} \nabla) \mathbf{u} &= \sigma(r) \delta(z) \left(u_r \frac{\partial}{\partial r} + u_z \frac{\partial}{\partial z} + u_\phi \frac{\partial}{r \partial \phi} \right) (\mathbf{e}^{(r)} u_r + \mathbf{e}^{(z)} u_z + \mathbf{e}^{(\phi)} u_\phi) = \\ &= \sigma(r) \delta(z) \left[\left(u_r \frac{\partial u_r}{\partial r} - \frac{u_\phi^2}{r} \right) \mathbf{e}^{(r)} + u_r \left(\frac{\partial u_\phi}{\partial r} + \frac{u_\phi}{r} \right) \mathbf{e}^{(\phi)} \right]. \end{aligned} \quad (36)$$

When deriving (36), we took into account azimuthal symmetry, the equality $u_z = 0$, and the relations $\partial \mathbf{e}^{(r)} / \partial \phi = \mathbf{e}^{(\phi)}$, $\partial \mathbf{e}^{(\phi)} / \partial \phi = -\mathbf{e}^{(r)}$. Integrating (36), we obtain

$$\int_{-z_0}^{z_0} \rho_d (\mathbf{u} \nabla) \mathbf{u} dz = \sigma(r) \left[\left(u_r \frac{\partial u_r}{\partial r} - \frac{u_\phi^2}{r} \right) \mathbf{e}^{(r)} + u_r \left(\frac{\partial u_\phi}{\partial r} + \frac{u_\phi}{r} \right) \mathbf{e}^{(\phi)} \right]. \quad (37)$$

If the magnetic field and the plasma velocity above the accretion disk ($z \geq 0$) are known, then the solution in the region under the disk continues in accordance with the rules

$$B_z \rightarrow B_z, \quad B_r \rightarrow -B_r, \quad B_\phi \rightarrow -B_\phi. \quad (38)$$

$$v_z \rightarrow -v_z, \quad v_r \rightarrow v_r, \quad v_\phi \rightarrow v_\phi. \quad (39)$$

Since the field components B_r , B_ϕ at the equator (for $z = +0$), this implies that the field line is discontinuous there.

Using relations (39), it can be shown that, in the

$$\int_{-z_0}^{z_0} \rho (\mathbf{v} \nabla) \mathbf{v} dz = 0.$$

Next, we obtain

$$\int_{-z_0}^{z_0} \nabla B^2 dz = 0, \quad (40)$$

since B^2 is a continuous function of z .

The term $(\mathbf{B} \nabla) \mathbf{B}$ will be written in the form

$$(\mathbf{B} \nabla) \mathbf{B} = \left(B_r \frac{\partial}{\partial r} + B_z \frac{\partial}{\partial z} + B_\phi \frac{\partial}{\partial \phi} \right) (B_r \mathbf{e}^{(r)} + B_z \mathbf{e}^{(z)} + B_\phi \mathbf{e}^{(\phi)}). \quad (41)$$

If the integration is taken over an infinitely small interval, a nonzero contribution will be obtained only from terms containing the derivative $\partial / \partial z$ (a derivative of a discontinuous function yields a δ function). After the integration, we obtain

$$\int_{-z_0}^{z_0} (\mathbf{B} \nabla) \mathbf{B} dz = 2 B_z (B_r \mathbf{e}_r + B_\phi \mathbf{e}_\phi). \quad (42)$$

The integration of the last term in (34) is trivial due to the presence of the δ function:

$$\int_{-z_0}^{z_0} \frac{\rho_d g \mathbf{R}}{R^3} dz = \frac{\sigma g}{r^2} \mathbf{e}^{(r)}. \quad (43)$$

Collecting these various relations, we obtain two equalities (for the r and ϕ components in the equation), which are a direct consequence of (34) in the model with an infinitely thin disk: (34) :

$$u_r \frac{\partial u_r}{\partial r} - \frac{u_\phi^2}{r} = \frac{2}{\sigma} B_r B_z - \frac{g}{r^2}, \quad (44)$$

$$\frac{u_r}{r} \frac{\partial}{\partial r} (r u_\phi) = \frac{2}{\sigma} B_\phi B_z. \quad (45)$$

The conservation of mass yields another relation:

$$\frac{1}{r} \frac{\partial}{\partial r} (r \sigma u_r) + 2 j_z = 0, \quad (46)$$

where $j_z = \rho v_z$ is the density of the z component of the mass flux. In (44)-(46), the magnetic-field components and plasma parameters are taken for the disk surface ($z = +0$). Equations (44)-(46) play the role of boundary conditions, which must be satisfied by the solution. They were derived from the equations of motion in differential form. Obviously, the same conditions can be obtained using the integral form of the equations of motion.

V. SELF-SIMILAR SOLUTIONS

We are primarily concerned here with the plasma dynamics immediately above the disk, at distances z much smaller than the radius of the disk R_{disk} . In the limit $z \ll R_{\text{disk}}$, only two parameters with the dimensions of length remain in the problem: z and r . Therefore, the solution will be self-similar in this limit [11]. Let us underscore an important feature of these solutions: they describe flows only at small distances from the disk, and are not applicable at distances comparable to or exceeding the size of the disk. Equations (4)-(7) have self-similar solutions of the form .

$$\left. \begin{aligned} \mathbf{v}(r, z, \phi) &= r^{-\delta_v} \tilde{\mathbf{v}}(z/r, \phi), \\ \rho(r, z) &= r^{-\delta_\rho} \tilde{\rho}(z/r), \\ \mathbf{B}(r, z, \phi) &= r^{-\delta_B} \tilde{\mathbf{B}}(z/r, \phi). \end{aligned} \right\} \quad (47)$$

The superscripts δ_v , δ_ρ , and δ_B are determined from the following conditions. Substituting (47) into (4) leads to the equations

$$2\delta_B - \delta_\rho = 2\delta_v = 1. \quad (48)$$

Equality (27) is satisfied under the condition

$$\delta_v + \delta_B = 1. \quad (49)$$

Thus,

$$\delta_v = \delta_B = \frac{1}{2}, \quad \delta_\rho = 0 \quad (50)$$

and hence,

$$\left. \begin{aligned} \mathbf{v}(r, z, \phi) &= r^{-1/2} \tilde{\mathbf{v}}(z/r, \phi), \\ \rho(r, z) &= \tilde{\rho}(z/r), \\ \mathbf{B}(r, z, \phi) &= r^{-1/2} \tilde{\mathbf{B}}(z/r, \phi). \end{aligned} \right\} \quad (51)$$

Note that it is possible to determine the subscripts unambiguously only for $u_r \neq 0$; i.e., when there is advection of the plasma toward the center of gravitation. When $u_r = 0$, one of the subscripts is arbitrary, so that there is a single-parameter family of selfsimilar solutions. A large number of such solutions have been studied previously (see, for example, [28] and references therein). For clarity, we will compare our results with those of [16], which is similar to our study in terms of the formulation for the wind. The case $\delta_\rho = 3/2$, $\delta_B = 5/4$. was considered in [16].

In frozen-in coordinates, the law of similarity (51) is described by the relation

$$\mathbf{R}(\alpha, \psi, \varphi) = \psi^{2/3} \tilde{\mathbf{R}}\left(\frac{\alpha}{\psi}, \varphi\right), \quad (52)$$

Therefore, the two-variable functions in (10) are expressed in terms of one-variable functions. Denoting $s = \alpha/\psi$, we can write

$$r(\alpha, \psi) = \psi^{2/3} \tilde{r}(s), \quad z(\alpha, \psi) = \psi^{2/3} \tilde{z}(s), \quad \eta(\alpha, \psi) = \tilde{\eta}(s). \quad (53)$$

Let us verify the equivalence of (51) and (53). Substituting (53) into (22), we derive the plasma density:

$$\rho(s) = \frac{3}{2} \frac{1}{\tilde{r}(\tilde{r}\tilde{z}_s - \tilde{z}\tilde{r}_s)}, \quad (54)$$

where the subscript s denotes differentiation with respect to s . This function depends only on s , or, if we switch to cylindrical coordinates, on the ratio z/r , as in (51). The components of the magnetic-field strength are

$$B_r = \psi^{-1/3} \rho \tilde{r}_s, \quad B_z = \psi^{-1/3} \rho \tilde{z}_s, \quad B_\phi = \psi^{-1/3} \rho \tilde{r} \tilde{\eta}_s. \quad (55)$$

We find for the components of the plasma velocity

$$v_r = \psi^{-1/3} \left(\frac{2}{3} \varepsilon \tilde{r} + (f - \varepsilon s) \tilde{r}_s \right), \quad (56)$$

$$v_z = \psi^{-1/3} \left(\frac{2}{3} \varepsilon \tilde{z} + (f - \varepsilon s) \tilde{z}_s \right), \quad (57)$$

$$v_\phi = \psi^{-1/3} \tilde{r} (\psi \Omega + (f - \varepsilon s) \tilde{\eta}_s). \quad (58)$$

The law of similarity $\mathbf{v} \sim r^{-1/2}$ will be satisfied if $f = \text{const}$, while Ω depends on ψ as follows:

$$\Omega(\psi) = \tilde{\Omega}/\psi. \quad (59)$$

represents the angular velocity of the selected field line in units of the Keplerian angular velocity. Substituting (52) into (32), we obtain a system of three ordinary second-order differential equations¹ for the functions $\tilde{r}(s)$, $\tilde{z}(s)$, and $\tilde{\eta}(s)$. Since second derivatives appear in the equations in linear form, the system can be solved for the second derivatives. For $\varepsilon \neq 0$, the right-hand sides of the equations depend explicitly on s ; f and s appear in the equations only in the combination $f - \varepsilon s$. The parameter s has an arbitrary zero point. Indeed, shifting the reference point for s via the substitution $s = s' + s_0$ is equivalent to redefining the constant f : $f \rightarrow f' = f - \varepsilon s_0$. We will take the surface of the accretion disk as the reference point for s : $\tilde{z}(s = 0) = 0$.

The resulting equations can be written as a system of six ordinary first-order differential equations. We introduce the six-dimensional vector ξ_i with the components

$$\xi_1 = \tilde{r}_s, \quad \xi_2 = \tilde{z}_s, \quad \xi_3 = \tilde{\eta}_s, \quad (60)$$

$$\xi_4 = \tilde{r}, \quad \xi_5 = \tilde{z}, \quad \xi_6 = \tilde{\eta}. \quad (61)$$

Then, for $i = 1, 2, 3$ the equations have the structure

$$\frac{d\xi_{1,2,3}}{ds} = \frac{N_{1,2,3}}{D}, \quad (62)$$

while for $i = 4, 5, 6$

$$\frac{d\xi_{4,5,6}}{ds} = \xi_{1,2,3}. \quad (63)$$

The denominator D is the determinant of a certain 3×3 matrix; it appears when the initial system of equations is solved relative to the highest derivatives. The functions N_i and D depend on ξ_1, \dots, ξ_5 ; ξ_6 does not appear in the equations due to the azimuthal symmetry of the problem. In addition, the right-hand sides of (62) depend on s . For fixed ψ and φ , the curve $\mathbf{R} = \psi^{2/3} \tilde{\mathbf{R}}(s, \varphi)$ specifies a magnetic-field line in parametric form. In

the case of self-similar solutions, it will suffice to find one field line; others can be derived using the similarity transformation (multiplication by $\psi^{2/3}$) and rotation about the z axis by some angle φ . The field line corresponding to the frozen-in coordinate $\psi = 1$ crosses the disk at the distance $r(0) = 1$. The value $\tilde{\eta}(0)$ can be arbitrary, for example, zero. The solution of the system of differential equations should then satisfy the initial conditions

$$\tilde{r}(0) = 1, \quad \tilde{z}(0) = 0, \quad \tilde{\eta}(0) = 0. \quad (64)$$

The frozen-in coordinate ψ and cylindrical coordinate r are related by the simple expression

$$r = \psi^{2/3}. \quad (65)$$

To obtain an unambiguous solution of the system of equations, we must have another three conditions.

It follows from (54) that the plasma density at the disk surface is $\rho = 3/(2\tilde{z}_s^{(0)})$. Therefore, the components of the magnetic field in the disk are

$$B_r^{(0)} = \frac{3}{2} \frac{\tilde{r}_s^{(0)}}{\tilde{z}_s^{(0)}} \psi^{-1/3}, \quad B_z^{(0)} = \frac{3}{2} \psi^{-1/3}, \quad B_\phi^{(0)} = \frac{3}{2} \frac{\tilde{\eta}_s^{(0)}}{\tilde{z}_s^{(0)}} \psi^{-1/3}, \quad (66)$$

where the superscript 0 denotes values for $z = +0$.

Let us write the boundary conditions (44)(46) for the self-similar solution. Substituting (65) into (31) and applying the initial conditions, we obtain the velocity of the accretion disk:

$$u_r = \frac{2}{3} \varepsilon \psi^{-1/3}, \quad u_\phi = \tilde{\Omega} \psi^{-1/3}. \quad (67)$$

¹ The equations are cumbersome, and we do not present them here. The analytical manipulations were performed using the Maple package.

The velocity component $u_r < 0$, so that ε should also be negative. Together with the plasma velocity, u_r and u_ϕ depend on r as $r^{-1/2}$. It follows from (44) or (45) that the mass surface density of the disk is a linear function of r :

$$\sigma = \mu r, \quad (68)$$

where $\mu = \text{const.}$ In (46), j_z is expressed using (54) and (56) as follows:

$$j_z = \frac{3}{2} f \psi^{-1/3}. \quad (69)$$

We derive from the continuity equation (46)

$$\mu = 3 f / |\varepsilon|. \quad (70)$$

Substituting (66), (67), and (68) into the boundary conditions (44) and (45), we obtain

$$\tilde{\Omega}^2 = 1 - \frac{2}{9} \varepsilon^2 + \frac{3 \varepsilon}{2 f} \frac{\tilde{r}_s^{(0)}}{\tilde{z}_s^{(0)}}, \quad (71)$$

$$\tilde{\eta}_s^{(0)} = -\frac{2}{9} f \tilde{\Omega} \tilde{z}_s^{(0)}. \quad (72)$$

This last expression can be used to calculate the azimuthal component of the magnetic field at the equator:

$$B_\phi^{(0)} = -\frac{1}{3} f \tilde{\Omega} \psi^{-1/3}. \quad (73)$$

We can see from these conditions that the rotational velocity of a field line is lower than the Keplerian velocity. Only in the limit $\varepsilon \rightarrow 0$ do we obtain $\tilde{\Omega} = 1$, which corresponds to a Keplerian accretion disk. Note that the boundary conditions in the disk fully specify the azimuthal magnetic field above the disk. Since ε does not appear in (72) and (73), these equalities should be satisfied even in the limit $\varepsilon \rightarrow 0$.

The conditions (44) and (46) make it possible to relate in a self-consistent way all the parameters of the accretion disk and of the outflowing plasma, and, in particular, to take into account the decrease in the surface density of the disk due to the expansion of the plasma. The equality (45) leads to the additional condition (72), which must be satisfied by the solution.

VI. SOLUTION IN THE VICINITY OF THE DISK

Let us consider an important limiting case, when the radial velocity of the disk is small compared to the Keplerian velocity, $|\varepsilon| \ll 1$. Let us also assume that the velocity of the plasma outflow from the disk is fairly low. Then, the following equations are valid in the vicinity of the disk:

$$\frac{d\tilde{r}_s}{ds} = \tilde{r}_s G, \quad \frac{d\tilde{z}_s}{ds} = \tilde{z}_s G, \quad \frac{d\tilde{\eta}_s}{ds} = \tilde{\eta}_s G. \quad (74)$$

Here,

$$G = \frac{1}{6 f^3} \frac{6 f \tilde{z}_s (3 \tilde{r}_s \delta \tilde{r} - \tilde{z}_s \tilde{z}) + 9 \varepsilon \tilde{r}_s^2 - 2 \varepsilon f \tilde{z}_s \tilde{\eta}_s}{\tilde{z}_s (\tilde{r}_s^2 + \tilde{z}_s^2 + \tilde{\eta}_s^2)}, \quad (75)$$

where $\delta \tilde{r} = \tilde{r} - 1$. To derive (74), $\tilde{\Omega}$ in the form (71) is substituted into the exact equations (62) and expanded in small parameters. Equations (74) are valid when $\delta \tilde{r}$ and \tilde{z} are small compared to unity and

$$f^2 \tilde{r}_s \ll 1, \quad f^2 \tilde{z}_s \ll 1, \quad f^2 \tilde{\eta}_s \ll 1. \quad (76)$$

These conditions follow from the requirement that subsequent terms of the expansion be small. It follows from (74) that

$$\xi_s = \xi_s^{(0)} \exp \left(\int_0^s G ds \right), \quad (77)$$

where ξ_s is any one of the three functions \tilde{r}_s , \tilde{z}_s , or $\tilde{\eta}_s$. After integrating (77) with the initial conditions $\tilde{z}(0) = 0$, $\delta\tilde{r}(0) = 0$, $\tilde{\eta}(0) = 0$, we find that a field line is straight in the vicinity of the disk:

$$\delta\tilde{r}(s) = k_r \tilde{z}(s), \quad \tilde{\eta}(s) = k_\eta \tilde{z}(s), \quad (78)$$

where k_r and k_η are constant. As we can see from (72), $k_\eta = -2f/9$ in the approximation considered. k_r specifies the direction of the poloidal field in the vicinity of the disk. Using equality (78), the second of Eqs. (74) can be presented in the form

$$\frac{d^2 \tilde{z}}{ds^2} = w^2 \tilde{z} - |\varepsilon| \zeta, \quad (79)$$

where

$$w^2 = \frac{1}{f^2} \frac{3k_r^2 - 1}{k_r^2 + k_\eta^2 + 1}, \quad (80)$$

$$\zeta = \frac{1}{6f^3} \frac{9k_r^2 - 2fk_\eta}{k_r^2 + k_\eta^2 + 1}. \quad (81)$$

When $w^2 < 0$, (79) has an oscillating solution, which is physically unacceptable. The inequality $k_r^2 > 1/3$, obtained in [16], must be satisfied by field lines moving away from the disk. This implies that a magnetic-field line must be inclined to the disk at an angle smaller than 60° .

Unlike [16], we also obtained another constraint. When $w^2 > 0$, the solution (79) has the form

$$\tilde{z} = \frac{|\varepsilon| \zeta}{w^2} (1 - \cosh(ws)) + \frac{\tilde{z}_s^{(0)}}{w} \sinh(ws), \quad (82)$$

where $\tilde{z}_s^{(0)}$ is an arbitrary constant equal to the derivative \tilde{z}_s in the disk. It follows from (82) that solutions cannot exist for arbitrarily small $\tilde{z}_s^{(0)}$. The field line will move away from the disk only when

$$\tilde{z}_s^{(0)} > \tilde{z}_s \min = \frac{|\varepsilon| \zeta}{w}. \quad (83)$$

This is equivalent to a constraint on the z component of the plasma outflow velocity:

$$v_z|_{z=0} > v_z \min = f \frac{|\varepsilon| \zeta}{w}. \quad (84)$$

The nature of this constraint, which appears when $\varepsilon \neq 0$, is easy to understand. In this case, each particle of the plasma moves along its orbit with a velocity that is smaller than the Keplerian velocity. In the z direction, the particle is in the effective potential

$$U(\tilde{z}) = -\frac{1}{2} w^2 \tilde{z}^2 + |\varepsilon| \zeta \tilde{z}.$$

In order to be ejected from the disk, the particle must overcome the potential barrier corresponding to the difference between the gravitational and centrifugal forces.

The plasma density in the vicinity of the disk is

$$\rho = \frac{3}{2\tilde{z}_s} = \frac{3}{2\left(z_s^{(0)} \text{ch}(ws) - \frac{|\varepsilon| \zeta}{w} \text{sh}(ws)\right)}. \quad (85)$$

In the disk itself, it is finite:

$$\rho|_{z=0} = \frac{3}{2\tilde{z}_s^{(0)}} < \frac{3w}{2|\varepsilon| \zeta}. \quad (86)$$

In the limit $\varepsilon \rightarrow 0$, $\tilde{z}_s^{(0)} \rightarrow 0$, the density in the disk increases without bound, as in [16].

VII. PASSAGE THROUGH CRITICAL SURFACES

The general solutions of the equations describing stationary MHD flows are singular on certain surfaces, usually called critical surfaces. These have already been found for one-dimensional flows, as was proposed by Parker for the solar wind [33]. In one dimensional solutions, they are manifest as critical points, while, in the three-dimensional case, a set of critical points forms a surface. In the one-dimensional solutions, the critical points coincide with sonic points, where the local velocity of weak perturbations becomes comparable to the flow velocity, while the type of equation changes from elliptical to hyperbolic and back. Even the earliest solutions obtained for axisymmetrical flows indicated that this is not the case in general [16]. The nature of critical surfaces and their role in the formation of stationary MHD solutions was investigated in [34, 35]. It turns out that critical surfaces divide regions of solutions with different causeeffect relations, and these surfaces do not, in general, coincide with the surfaces where the form of the equation changes.

Critical surfaces as applied to self-similar solutions were studied in [36]. The parameters of critical surfaces for self-similar flows with $E_\varphi \neq 0$ were studied in detail in [27]. In these solutions, the locations of the critical surfaces are specified by the zeros of the denominator D in (62). In the general case (with a nonzero wind temperature), the denominator vanishes at the slow magnetoacoustic separatrix surface, Alfvén surface, and fast magnetoacoustic surface. Regularizing the solution on these surfaces makes it possible to determine the density of the plasma flux from the disk and the two tangential components of the magnetic field at the disk surface (the normal component is specified by the conditions adopted for the problem). In the case of a cool plasma, the slow magnetoacoustic velocity is zero, and one of the regularization conditions disappears. Therefore, the plasma flux from the disk surface must be specified for a cool plasma. This means that the ratio of the density of the plasma flux from the disk and the z component of the magnetic field is also specified. A similar approach was used in [16] (where k is analogous to f).

Thus, in self-similar flows of cool plasma, regularizing the solution on the Alfvén and fast magnetoacoustic critical surfaces specifies both the azimuthal and tangential components of the magnetic field based on the flow. One characteristic feature of self-similar solutions is that, as a rule, it is not possible to regularize them on the fast magnetoacoustic critical surface, only at the Alfvén surface. Recently, Vlahakis et al. [37] were able to regularize a self-similar solution on the fast magnetoacoustic surface, but, even in this case, the solution disappears very rapidly behind this surface. Therefore, as in [16], we will not try to regularize the solution on the fast magnetoacoustic surface. As a result, one of the components of the magnetic field (the azimuthal component in [16]) remains free and must be specified a priori. The other component is determined from the solution and the regularization condition at the Alfvén surface. The denominator D in (62) contains the factor

$$e_1 \equiv \rho - (f - \varepsilon s)^2. \quad (87)$$

In the disk, $e_1 > 0$ when $f^2 \tilde{z}_s^{(0)} < 3/2$. Since the density decreases along a field line, e_1 vanishes at some point on the Alfvén surface. To keep the solution finite, the functions N_i must also vanish at this point. After substituting $\rho = (f - \varepsilon s)^2$ into N_i , all the functions N_i possess the common factor

$$\begin{aligned} e_2 \equiv & \left(-108 \tilde{\Omega} b + \left(12 \tilde{r}^2 \varepsilon \tilde{\Omega} \tilde{z} - 27 \tilde{\eta}_s \right) b^2 \right) \tilde{r}_s^2 + \\ & + \left(\left(8 \tilde{r}^3 \tilde{z} \varepsilon^2 \tilde{\eta}_s + 8 \tilde{r} \tilde{z}^3 \varepsilon^2 \tilde{\eta}_s + 12 \tilde{r} \tilde{z}^2 \varepsilon \tilde{\Omega} \tilde{z}_s - 12 \tilde{r}^3 \varepsilon \tilde{\Omega} \tilde{z}_s + 36 \tilde{r}^3 \tilde{z} \tilde{\Omega}^2 \tilde{\eta}_s \right) b^2 - 108 \frac{\tilde{\Omega} b \tilde{z} \tilde{z}_s}{\tilde{r}} \right) \tilde{r}_s + \\ & + \left(-36 \tilde{r}^4 \tilde{z}_s \tilde{\Omega}^2 \tilde{\eta}_s - 8 \tilde{r}^4 \tilde{z}_s \varepsilon^2 \tilde{\eta}_s - 12 \tilde{r}^2 \tilde{z} \varepsilon \tilde{\Omega} \tilde{z}_s^2 - 8 \tilde{r}^2 \tilde{z}_s \varepsilon^2 \tilde{\eta}_s \tilde{z}^2 - 27 \tilde{r}^2 \tilde{\eta}_s^3 - 27 \tilde{\eta}_s \tilde{z}_s^2 \right) b^2 + \\ & + 54 \frac{\tilde{\eta}_s}{\sqrt{\tilde{r}^2 + \tilde{z}^2}} - 108 b \tilde{\Omega} \tilde{r}^2 \tilde{\eta}_s^2, \end{aligned} \quad (88)$$

where $b = f - \varepsilon s$. Therefore, the condition that the denominator and three functions N_i in (62) simultaneously vanish results in the system of two equations

$$e_1 = 0, \quad e_2 = 0, \quad (89)$$

which can be solved analytically for \tilde{r}_s and \tilde{z}_s . Consequently, the derivatives $(\tilde{r}_s, \tilde{z}_s)$ can be taken to be specified at the critical point (\tilde{r}, \tilde{z}) .

VIII. SOLUTION IN THE LIMIT $\varepsilon \rightarrow 0$.

It is of interest to consider the solution in the limit of very slow accretion, when $\varepsilon \rightarrow 0$. This case is of interest because the disk becomes Keplerian in this limit ($\tilde{\Omega} = 1$), so that our results can easily be compared to those of [16], and the physics of accretion made possible by a wind outflow becomes particularly simple.

In the case of small ε , εs can be neglected compared to f . The parameter s then disappears from the right-hand sides of (62), and the simplified equations have two integrals of motion: the momentum L and the energy W . For the self-similar solution,

$$L = \tilde{r}^2 \left(f \tilde{\Omega} - \tilde{\eta}_s (\rho - f^2) \right), \quad (90)$$

$$W = \frac{1}{2} f^2 (\tilde{r}_s^2 + \tilde{r}^2 \tilde{\eta}_s^2 + \tilde{z}_s^2) - \frac{1}{2} \tilde{\Omega}^2 \tilde{r}^2 - \frac{1}{\sqrt{\tilde{r}^2 + \tilde{z}^2}}, \quad (91)$$

where we can assume $\tilde{\Omega} = 1$ on the right-hand sides of the equality.

Substituting the values at the disk surface into (90) and using (64) and (72), we find

$$L = \frac{4}{3} f \left(1 - \frac{1}{6} f^2 \tilde{z}_s^{(0)} \right). \quad (92)$$

Since $\tilde{z}_s^{(0)} \geq 0$, $L \leq L_{\max} = \frac{4}{3} f$. This new important constraint on the angular-momentum flux carried away from the disk follows from the boundary conditions at the disk. It means that a relatively small flux of angular momentum must be carried away by the wind to enable accretion. In addition, the boundary conditions at the disk specify the azimuthal component of the magnetic field, and the problem is fully specified even when the solution is regularized at one Alfvén point.

As we can see from (84), in the limit $\varepsilon \rightarrow 0$, the initial velocity of the plasma outflow can be arbitrarily small. In this case, the energy tends to the minimum $W_{\min} = -3/2$ [see (91)]. Assuming that the conditions (76) are satisfied at the disk surface, we obtain $L = 4/3 f$, $W = -3/2$. Therefore, at the Alfvén point,

$$\tilde{r}^2 = \frac{L}{f} = \frac{4}{3}. \quad (93)$$

The energy integral and Eqs. (89) can be used to express analytically the derivatives \tilde{r}_s , \tilde{z}_s , and $\tilde{\eta}_s$ at the Alfvén point in terms of \tilde{z} . A straightforward but fairly cumbersome analysis shows that the solutions (89) exist for $L = 4/3 f$, $W = -3/2$ if

$$f > \frac{9}{16} \sqrt{6(5 + 3\sqrt{3})} \approx 4.4. \quad (94)$$

\tilde{z} must be confined within the interval

$$z_{\min} \leq \tilde{z} \leq z_{\max}, \quad (95)$$

where

$$z_{\max} = \left[\frac{216f\sqrt{16f^2 - 45} - 736f^2 - 1215}{1200f^2} \right]^{1/2}, \quad (96)$$

$z_{\min} = 0$ when $f < 8.27$, and

$$z_{\min} = \frac{2}{\sqrt{3}} \frac{\sqrt{2351f^4 - 159894f^2 - 59049}}{329f^2 + 243} \quad (97)$$

when $f > 8.27$. It follows from (96) that $\tilde{z} < 0.327$ at the Alfvén point; this means that the Alfvén point cannot be far from the disk surface.

We solved (62) and (63) in the limiting case $\varepsilon \rightarrow 0$ numerically using standard techniques. We started from the Alfvén point, with f as the input parameter. As the initial condition, we took the Alfvén point with $\tilde{r} = 2/\sqrt{3}$ and \tilde{z} from the interval (95); the derivatives were selected using the described procedure. At the Alfvén point, the

right-hand sides of (62) were determined using l'Hopital's rule. The initial value for \tilde{z} was selected so that the field line crossed the disk at the point $\tilde{r} = 1$. We adopted a value slightly larger than $-3/2$ for W . The reason for this is that, when $W = -3/2$, the plasma velocity at the disk surface vanishes and the plasma density at the disk displays a nonintegrable singularity: $\rho = 3/(2w\tilde{z})$. Therefore, we chose W so that $\tilde{z}_s^{(0)} \sim 10^{-3}$ in the disk, or, in other words, so that the velocity of the plasma outflow relative to the disk was of the order of $f \cdot 10^{-3}$ of the Keplerian velocity. The requirement that $\tilde{z}_s^{(0)}$ be nonzero also results from the fact that the inequality (84) should be rigorous.

Figure 4 presents the f dependence of \tilde{z} at the Alfvén point together with z_{\min} and z_{\max} . The minimum f for which the solution exists is $f_{\min} = 5.28$. $z_{\max} = z_A$ when $f = f_{\min}$.

Figure 5 presents a line of the poloidal magnetic field for $f = 6$, which does not differ much from f_{\min} . $k_r = 0.96$, so that the line is inclined to the equator at an angle that is close to $\pi/4$; at the disk surface, $\tilde{z}_s = 10^{-3}$. There is no significant acceleration of the plasma; at the Alfvén point, $\tilde{z}_s = 0.033$ and $\tilde{r}_s = 0.010$, so that the components of the plasma velocity v_z and v_r are small compared to the Keplerian velocity. Figure 6 presents the plasma density on a field line for various heights. In the case of motion from the disk toward the Alfvén point, s varies from 0 to $s_{\max} \approx 30$. Therefore, εs can be neglected compared to f when

$$|\varepsilon| \ll f/s_{\max}. \quad (98)$$

This condition, as well as the inequality (76), ensures the applicability of the approximation used in the case considered. The analytical solution (82) coincides with the numerical solution along nearly the entire field line, with the exception of the immediate vicinity of the Alfvén point.

IX. DISCUSSION

The conclusion that the plasma outflow from an accretion disk can exert a substantial effect on the dynamics of the accretion disk is not new [19]. The originality of our work is our consideration of the case when plasma falls onto the gravitating center along with the frozen-in magnetic field, rather than seeping across magnetic-field lines (see, for example, [21] and references therein). Our results indicate that dissipationless disk accretion of the type considered is, indeed, possible, as is demonstrated by the derived self-consistent solution. The mechanism for carrying away angular momentum that we have considered should be taken into account in studies of accretion in sources displaying violent ejections of matter. This is particularly important for our understanding of the processes occurring in a number of peculiar sources, prominently represented by SS433 [2]. Objects in which disk accretion occurs primarily due to the carrying away of angular momentum by an outflowing wind can have bolometric luminosities that are appreciably lower than the kinetic-energy flux in the outflowing wind. This is what leads to the most important difficulties in explaining the plasma-ejection mechanism in both SS433 and young stellar objects. The derived self-consistent solution possesses a number of interesting properties. The boundary conditions (71), (72) fully specify the azimuthal magnetic field in the disk. In this case, with fixed f and L , the toroidal and tangential components of the magnetic field are determined unambiguously, unlike in the study of Blandford and Payne [16]. The flow turns out to be fully specified. This begs the question of whether there exists the fundamental possibility of passing through the fast magnetoacoustic critical point, since, at first glance, no free parameters remain. In fact, there is still one free parameter: ε , which specifies the radial velocity of the plasma in the disk. This parameter can be used to regularize the solution at the fast magnetoacoustic critical point. If this is possible, the entire solution will prove to be unambiguously specified. An unexpected feature of the obtained solutions is that, in the limiting case of dissipationless accretion, 100% of the infalling matter is ejected back out from the disk. In this respect, our results are similar to those of Lery et al. [38], who also considered dissipationless accretion, but without the formation of a disk. Both of these results indicate that, in any case, dissipation is probably unavoidable, to provide for the infall of some fraction of the matter onto the gravitating center. On the other hand, this feature of the flow may well be due to its self-similarity. Additional studies are required before we will be able to draw final conclusions. A natural extension of our study is the consideration of accretion due simultaneously to matter outflow from the disk and dissipation. Even at this stage, it should be possible to determine the relationship between the disk luminosity and the flux of outflowing matter, which can be compared to observations.

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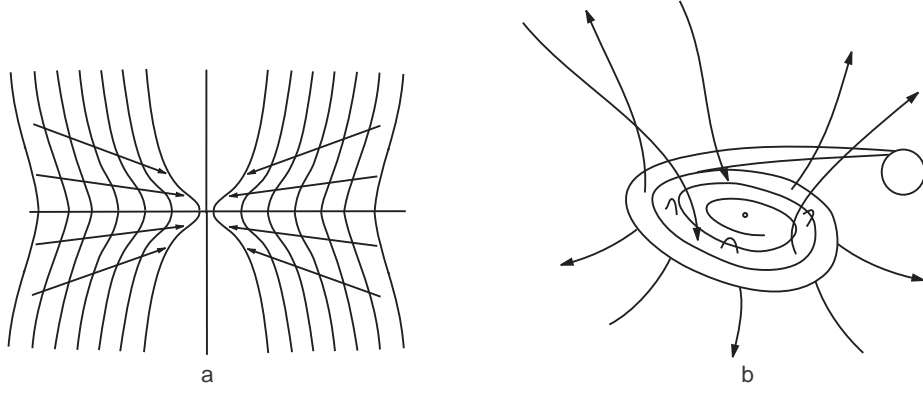


Figure 1: Role of the magnetic field in different formulations of the problem. Left: plasma diffuses across the magnetic-field lines. Right: plasma falls onto the gravitating center along the magnetic field. The wind flowing from the disk extends some field lines to infinity and carries away some fraction of the angular momentum of the infalling material.

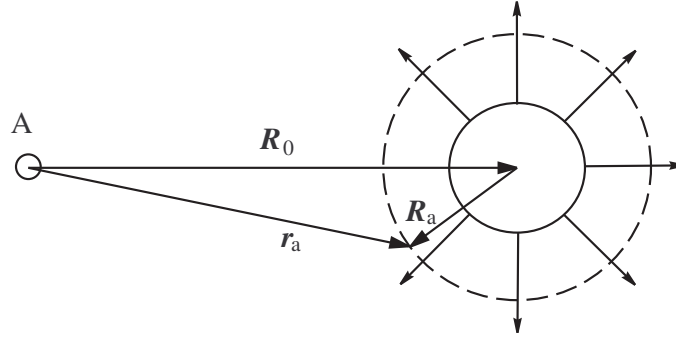


Figure 2: Loss of angular momentum via a magnetized wind. The gravitating center is at the point A.

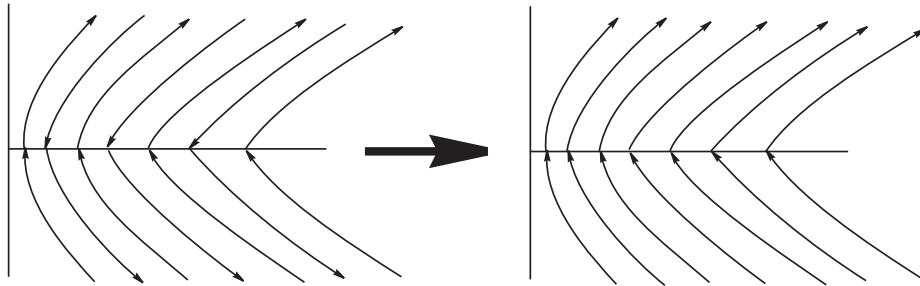


Figure 3: Independence of the plasma dynamics of the direction of the magnetic-field lines in an ideal MHD approximation. The left panel shows a realistic structure for the flux of the poloidal magnetic field; the total flux of the poloidal magnetic field is zero. In the right panel, the direction of the field lines has changed, while the plasma dynamics remain unaltered.

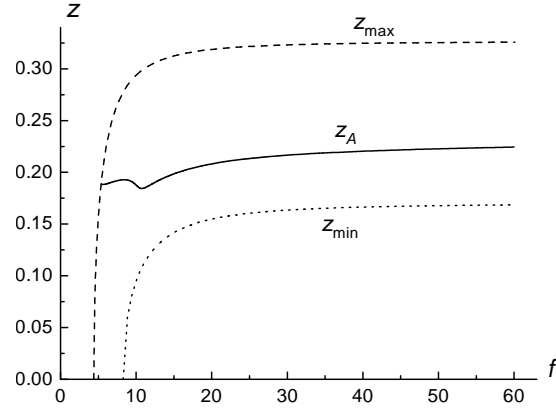


Figure 4: Values z_A of $\tilde{z}(f)$ for which a field line emerging from the Alfvén point crosses the equator when $\tilde{r} = 1$. z_{\max} and z_{\min} , which appear in (95), are also indicated.

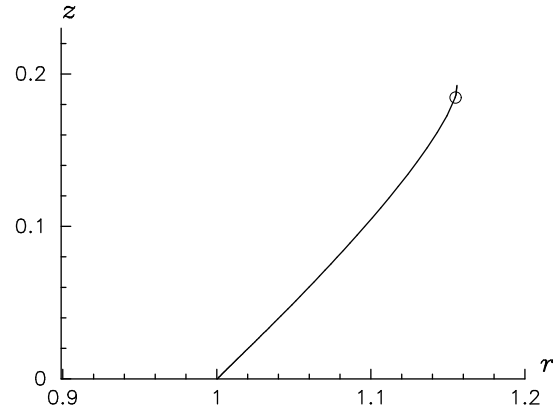


Figure 5: A line of the poloidal field for $f = 6$. The circle marks the position of the Alfvén singularity.

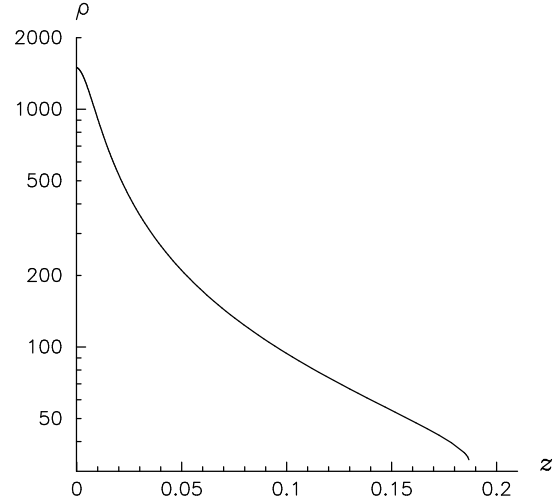


Figure 6: z dependence of the plasma density on the field line shown in Fig. 5.